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ABSTRACT

This paper discusses goal programming, a computer-based operations research technique that is basically a modification and extension of linear programming. The authors first discuss the similarities and differences between goal programming and linear programming, then describe the limitations of goal programming and its possible applications for educational planning and problem-solving. Most of the paper is devoted to demonstrating the formulation of linear programming and goal programming models and to presenting three detailed examples of how the goal programming model can be applied to solving different types of educational planning problems. These examples include scheduling algebra instruction for 60 high school students, designing a school district busing plan, and determining salary differentials for school district supervisory personnel under collective bargaining. (JG)

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APPLICATIONS OF GOAL PROGRAMMING TO EDUCATION

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Goal programming is both a modification and extension of linear programming (Lee, 1972). Linear programming is the name given to the operations research technique wherein a mathematical model of a problem situation is formulated to contain a linear objective function and constraints. Through the use of an iterative algorithm known as the simplex technique or a modification thereof, the objective function and its components, homogeneous choice variables, are optimized (maximized or minimized) subject to the constraints (limited resources or restrictions) stated in the model. Constraints represent relationships between the choice variables and are stated as linear inequalities and/or equalities. Because of the availability of computer programs for performing the mathematical calculations necessary to the solution of complex linear programming problems, it is not the computation but rather the model formulation that is the chief concern of the problem-solver.

The goal programming approach was originally formulated by Charnes, Cooper and Ferguson (1955) and named in Volume I of Management Models and Industrial Applications of Linear Programming (1961). The method presented

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used a weighted objective function. Ijiri (1965) presented an algoritm for goal programming in which the objective function is prioritized rather than weighted. This causes all resources in a problem to be applied to the first goal until it is satisfied and then the second goal until it is satisfied and so on. By 1968 Charnes and Cooper et. al., had made applications in the fields of media planning and manpower planning. In the last six years the number of articles concerning goal programming has increased greatly, and one book in particular (Lee, 1972) provides an excellent introduction to the area. Lee presented an algorithm for goal programming that is a modification of the standard linear programming simplex method. The computational basis of the examples discussed here depend on Lee's algoritm and his Fortran program. His program has also been adapted for time-sharing on the H-P (access) system at the University of Iowa.

How Goal Programming and Linear Programming are Alike

1. Both require the formulation of a model for transforming a real-world decision problem into a prescribed format.
2. Both are concerned with goal or objective achievement.
3. Both have the following model characteristics:
   a. The optimization of the objective function subject to a set of constraints or limited resources.
   b. Variables must have the property of non-negativity.
   c. Constraints and objective function are linear, i.e., all variables are to be the first power or "of the first order". No quadratic or higher order relationships may be included in the model formulation.

4. Both represent a systematic attempt toward rationality in decision-making.

5. Both are adaptable to analyzing decisions. When a computer program is available to perform the necessary calculations, the educational manager can easily "try-out" different formualtions of the problem model. He can, for example, observe the effects upon the solution of changing the coefficients of constraint variables.

How Goal Programming is Unlike Linear Programming

1. Goals -- The goals of a particular problem are modeled as constraints although they may be statements (written in the same format as constraints) which are not restrictive or descriptive of limited resources but positive in nature, representing a desirable condition. These constraint/goals will be hereafter referred to simply as goals. Since it is not usually necessary that each goal in the model be achieved exactly, goal programming allows for the likelihood that goals in a real-world problem may be conflicting. The deviational variables preserve the equality of each goal when combinations of goals are conflicting. Although the choice variables within goals must be consistent as to units of measure, goals may represent incommensurable quantities.

2. Deviational variables -- A feature of goal programming models not found in linear programming models is the use of deviational variables. These variables enable all goals to be stated as equalities. Of primary concern are the variables known as slack and surplus deviational variables. After all goals which are to be incorporated in the model are

identified, each must be assigned slack and/or suplus variables.

3. Objective function -- In goal programming the objective function usually contains no choice variables, but rather is made up of the deviational variables contained within the goals. When multiple goals are thus represented in the objective function, it is said to be multidimensional. Because the optimal solution would be the one in which the sum of the deviations from goals is minimal, the objective function is always minimized. In order that goals be achieved according to their importance, the deviational variables in the objective function must be prioritized according to the ordinal ranking of goals or the importance of each goal to the manager. The same priority level may be assigned to two or more deviational variables. Deviational variables on the same priority level may be weighted; it is perhaps most desirable to weight such variables when it makes clearer that the "cost" of underachievement of a particular goal is greater or lesser than another of equal importance. The units of measure of the deviational variables within the objective function may be nonhomogeneous, e.g., representing dollars and weeks, rather than one type of unit.

Limitations of Goal Programming

Lee (1972) indicates four limitations of goal programming as: (1) proportionality, (2) additivity, (3) divisibility and (4) deterministic. Proportionality, as a limitation, means that the linear relationships in the problem model must be proportional. Additivity indicates that the activities expressed in the objective function and goals must be additive in order to ensure linearity. Divisibility means that the values for decision variables

in the optimum solution of a goal programming problem can be nonintegral. Recently, a study on integer goal programming has appeared (Keown and Lee, 1975), therefore it is not anticipated that the application of goal programming will long be limited to a feasible solution set of positive real numbers. The deterministic nature of the goal programming model means that model coefficients must be constants. In this sense, the goal programming procedures is not better than most other rational procedures which require a "snapshot" of a continually changing world. Again, there is reason to believe that this limitation may be eliminated or at least reduced, in light of recent work showing that constraints may have variable limits (Sweeney and Williams, 1974) or represent a unique probability distribution (Contini, 1968).

Despite its present limitations, goal programming is believed to be applicable to a wide range of educational problems.

Educational Application of Goal Programming

As an extension of linear programming, it is assumed that goal programming can readily be adapted to the solution of educational problems previously utilizing linear programming techniques, but it appears that a greater value of goal programming lies in its facility for providing a more realistic model of the decision environment than has previously been possible with linear programming.

Linear programming has been utilized in the solution of such educational problems as:

1. minimizing travel distance in busing for integration (Ontjes, 1971).
2. maximizing the district-wide assignment of teachers (Berrie, 1972).
3. optimizing various aspects of a foundation type of state aid program

(Matzke, 1971).

4. determining university faculty salaries (Hartley, 1973).
5. designing alternative forms of salary schedules for public school teachers below the university level (Bruno, 1969, 1970).

Goal programming models have been created for the solutions to such educational problems as:

1. allocation of resources in institutions of higher learning (Lee, 1972).
2. determining a job factor compensation plan in a public school setting (Gunderson, 1975).

While applications of goal programming in education have been relatively scarce, there is reason to believe that it will prove to be a valuable decision-making aid to the school administrator once programs are widely available (Gunderson, 1975).

Model Formulation

The basic linear programming problem is formulated as follows:

Optimize $Z = \sum_{j=1}^{n} c_j x_j$

such that $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$

and $x_j \geq 0$

where a_{ij}, b_i, and c_j are arbitrary constants.

The basic goal programming problem is formulated as follows:

Min. $Z = \sum_{i=1}^{n} (d_i^- + d_i^+)$,

such that $\bar{A}x + \bar{I}d_i^- - \bar{I}d_i^+ = \bar{B}$

and $x, d^-, d^+ \geq 0$

where $\bar{A}$ is a n x n matrix, $\bar{I}$ is a n x n identity matrix and $\bar{B}$ is a n component column vector.

As in any model formulation the following steps should be taken:

1. define the variables and constraints.
2. formulate the constraint equations.
3. develop the objective function.

Summary

Goal programming is an extension and modification of linear programming which allows the educational manager to more closely simulate real-life situations. Both linear and goal programming are optimization techniques which lend themselves to increasing the rationality of decision-making. The foremost value of goal programming is in its facility for solving problems with hierarchically arranged, conflicting goals. While there are presently certain limitations of goal programming which may slightly narrow the scope of its feasibile applications, it is believed that its potential for educational problem solving is vast.

Examples of Educational Problems and Goal Programming Solutions

In the following three sections, examples of goal programming applications to educational problems are given. The reader will find that, while all three examples are simplified, the complexity of the given model formulations increases substantially from one model to the next.

1. Scheduling Instruction

Problem

In a high school, 60 students are enrolled in algebra. Those students can be taught through large group instruction (all 60 in a class), medium-sized group instruction (30 to a class), small group instruction (15 to a

class) or individual instruction. We need to decide how much time they should spend in each type of instruction, subject to certain conditions:

Condition 1. Regulations require that each student spend at least 250 minutes per week in algebra class or individual instruction. But, we also want to avoid having students spend more than 250 minutes per week in algebra. Letting T_L stand for the number of minutes per week each student will spend in large group instruction and similarly for medium group (T_M), small group (T_S) and individual instruction (T_I), this may be expressed by:

$$T_L + T_M + T_S + T_I + d_1^- - d_1^+ = 250$$

where d_1^- is the number of minutes per week less than 250 that each student spends in algebra and d_1^+ is the number of minutes per week more than 250 each student spends in algebra.

Condition 2. Due to limited space for large classes to meet, we would like to schedule algebra students for not more than 60 minutes per week of large group instruction. This is expressed by:

$$T_L - d_2^+ = 60$$

where d_2^+ is the number of minutes per week over 60 scheduled for large group instruction in algebra.

Condition 3. We would like to schedule each student for at least 40 minutes per week of small group instruction. This is expressed by:

$$T_S + d_3^- = 40$$

where d_3^- is the number of minutes per week less than 40 that each student is scheduled into small group instruction.

Condition 4. We would like each student to have at least 10 minutes per week of individual instruction. This is expressed by:

$$T_L + d_4^- = 10$$

where d_4^- is the number of minutes per week less than 10 that students spend in individual instruction.

Condition 5. We would like to limit the amount of teacher time used to teach algebra to 1,070 minutes per week. This is expressed by:

$$T_L + 2T_M + 4T_S + 60T_I - d_5^+ = 1070$$

where d_5^+ is the teacher time used in excess of 1,070 minutes per week.

Priorities

Now priorities must be established for the conditions -- actually for the deviation variables, the d's. Let us place the highest priority on each student having at least 250 minutes of instruction per week. This is expressed as $P_1 d_1^+$. Let us say that our second priority is to use not more than 1,070 minutes of teacher time per week. This is expressed as $P_2 d_5^+$. Similarly, priorities are established for the other deviation variables. These made up our object function which is expressed by:

$$\text{Minimize} = P_1 d_1^- + P_2 d_5^+ + P_3 d_2^+ + P_4 d_1^+ + P_5 d_4^- + P_6 d_3^-$$

Model

Thus the goal programming model for this problem is:

$$\text{Minimize } z = P_1 d_1^- + P_2 d_5^+ + P_3 d_2^+ + P_4 d_1^+ + P_5 d_4^- + P_6 d_3^-$$

$$T_L + T_M + T_S + T_I + d_1^- - d_1^+ = 250$$

$$T_L - d_2^+ = 60$$

$$T_S + d_3^- = 40$$

$$T_I + d_4^- = 10$$

$$T_L + 2T_M + 4T_S + 60T_I - d_5^+ = 1070$$

Solution

Solving the model yields the following results:

Type of Instruction	Minutes/Week
Large Group (T_L)	60
Medium Group (T_M)	155
Small Group (T_S)	25
Individual (T_I)	10

With that solution, students spend exactly 250 minutes per week in algebra and exactly 1070 minutes per week of teacher time is used. Thus all conditions except the one having to do with the minutes per week for small group instruction are met. Only 25 minutes per week are allocated to small group instruction rather than the 40 we wanted. That, however, was our lowest priority.

2. Busing

This example will deal with the busing of students to achieve specified percentages (or better a range of percentages) of students in schools by groups (such as race, sex, vocational interest, etc.). There has been a great deal of work done on this problem using a linear approach (Stimson and Thompson, 1974). Here a goal programming formulation of a busing problem and the solution for a sample problem will be presented.

The basic problem can be viewed as having two requirements. The first is to achieve a specified percentage range composition by group. The other is to minimize transportation costs via minimization of total busing distance.

The sample problem will be constructed as follows. We will assume a community with three schools and three corresponding tracts that provide students for the three schools respectively. The student population by group,

the school capacities, and the busing distances (using an average distance) are summarized in Tables 1 and 2.

Table 1

Tract	STUDENTS Group 1	Group 2	School Capacity
1	450	225	750
2	600	0	1000
3	50	700	650
Totals	1100	925	2400

Total Student Population: 2025

Table 2

Tract	Distances to Schools (in miles) 1	2	3
1	1.2	1.5	3.3
2	2.6	4.0	5.5
3	0.7	1.1	2.8

A linear programming formulation of this problem could only consider one objective - busing distance or percentages of students by groups. The goal programming formulation can consider both objectives. In the problem four priorities will be used. Priority one will be to have all students assigned to a school. Priority 2 will be to have no school assigned more students than its capacity will allow. Priority 3 will be to achieve a student

composition such that each group falls within a range of 40% to 60% of the total school population. And finally, priority 4 will be to minimize the total busing distance.

The problem can be summarized as follows where

X_{ijk} = the number of students from tract i in school j from group k.

School :

$$1 \quad X_{i1k} + d_1^- = 750$$

$$2 \quad X_{i2k} + d_2^- = 1000$$

$$3 \quad X_{i3k} + d_3^- = 650$$

These constraints force students to be assigned to all schools with no school filled beyond its student capacity.

Tract :

$$1 \quad X_{1j1} + d_4^- = 450$$

and

$$X_{1j2} + d_5^- = 225$$

$$2 \quad X_{2j1} + d_6^- = 600$$

and

$$X_{2j2} + d_7^- = 0$$

$$3 \quad X_{3j1} + d_8^- = 50$$

and

$$X_{3j2} + d_9^- = 700$$

These constraints force all the students to be assigned to a school.

Ratios for school:

$$1 \quad -0.6\Sigma X_{i11} + 0.4\Sigma X_{112} + d_{10}^- = 0$$

and

$$0.4\Sigma X_{i11} - 0.6\Sigma X_{112} + d_{11}^- = 0$$

$$2 \quad -0.6\Sigma X_{i21} + 0.4\Sigma X_{i22} + d_{12}^{-} = 0$$

and

$$0.4\Sigma X_{i21} - 0.6\Sigma X_{i22} + d_{13}^{-} = 0$$

$$3 \quad -0.6\Sigma X_{i31} + 0.4\Sigma X_{i32} + d_{14}^{-} = 0$$

and

$$0.4\Sigma X_{i31} - 0.6\Sigma X_{i32} + d_{15}^{-} = 0$$

These constraints establish the 40% to 60% range for each group of students.

Distance:

$$1.2\Sigma X_{11k} + 1.5\Sigma X_{12k} + 2.6\Sigma X_{13k} + 2.6\Sigma X_{21k} + 4.0\Sigma X_{22k} + 5.5\Sigma X_{23k} + 0.7\Sigma X_{31k} + 1.1\Sigma X_{32k} + 2.8\Sigma X_{33k} - d_{16}^{+} = 3800$$

This constraint forces the total distance bused beyond 3800 miles to be minimized. The value of 3800 miles was obtained as the solution to a linear programming transportation problem for the given data with the objective being to minimize total distance bused. Therefore for this goal programming problem, 3800 miles represent an ideal minimum distance.

Now according to stated priorities the objective function is

$$\text{Minimize } Z = P_1 \sum_{i=4}^{9} d_i^{-} + P_2 \sum_{i=1}^{3} d_i^{-} + P_3 \sum_{i=10}^{15} d_i^{-} + P_4 d_{16}^{-}$$

The solution is summarized in Table 3; it shows the number of students assigned from each tract and group to each school.

As can be seen priority 1 was met completely with all students assigned to a school. Priority 2 (filling all schools) was not met. There was a deviation of 375, but that is exactly the amount of excess capacity for the three schools. The desire to meet the 40% to 60% composition for each group was met exactly in schools one and three. School two had a 44% to 56% group

composition range which is still within the desired 40% to 60% range. Finally, priority 4 had a deviation of 295 which means the total busing distance for this situation is 4095 miles.

Table 3

School	Group	Tract 1	Tract 2	Tract 3	Group Totals	School Totals
1	1	0	450	0	450	750
	2	0	0	300	300	
2	1	450	0	50	500	900
	2	225	0	175	400	
3	1	0	150	0	150	375
	2	0	0	225	225	
Tract Totals		675	600	750		

3. Job Factor Compensation

This example is a summary of J.O. Gunderson's doctoral dissertation (1975). It concerns the development of a model for determining job factor compensation for supervisory personnel under collective bargaining. The basic idea is to distribute wages to supervisory personnel where the dollar amounts desired exceed the dollar amounts available. The situation models collective bargaining between the supervisory personnel and the board of education of a school district. The supervisor's jobs were assumed to be composed of twelve variables. The variables can be summarized as follows:

Variable	Definition
X_1	District advisory committee participation
X_2	Administration of different collective bargaining contracts
X_3	Scope of work
X_4	Budget development involvement
X_5	Supervision of multiple programs
X_6	Supervision of other supervisory staff
X_7	Supervision of classified staff
X_8	Responsibility for capitol equipment
X_9	Direct contact with students
X_{10}	Supervision of staff under a fiar dismissal law
X_{11}	Highest degree obtained
X_{12}	Years of supervisory experience

Thus each supervisor considered had a twelve item "job factor profile." This job factor profile determined the amount of special compensation an individual would receive beyond his base salary. Each variable was also scaled to reflect an internal hierarchy of importance within each factor. Each individual's salary was then the sum of his base salary plus the total dollar value of each variable times the individual's appropriate scale factor. The total budget for all supervisors' special compensation was the sum of the individual special compensation salaries. This is an important sum because limits placed on this total will affect the basic alignment between each individual's special compensation.

Six basic types of constraints were formulated to reflect the relationships between the variables. The first constraint was the total district resources allowed for special compensation. The second type were the variable

value constraints used to force equality among the variables unless the goal priorities affected them otherwise. The third type was the negotiation constraint and it was used to balance the total value of variables 1-6 against variables 7-12. Fourth was the factor sum constraint which was used to provide overall model factor consistency. The fifth and sixth types of constraints were used to control the scale hierarchy widths and midpoints respectively. The total number of constraints (rows) was 23. Rows 1-13 dealt with individual job factors and rows 14-23 dealt with the relations of job factors and personnel types to each other. After internal scaling of each variable there were 349 total factor weights.

Six priorities (goals) were established to reflect the overall view of how the job factor compensation model should work. Goal 1 was to use all of the resources allocated by the district for special compensation. The desired level was to limit underachievement of the total resources allocated. Goal 2 was to balance board initiated and supervisor initiated factors. Goal 3 was to balance the effect of the width and midpoint scale factors. Goal 4 was to maintain overall consistency of special compensation for the supervisors. Goal 5 was to maintain over all consistency and equality between each of the 12 variables. Goals 2-5 wanted then to limit both over and under achievement. Finally goal 6 was to allow the total resources spent to exceed the limit desired in Goal 1. That is, the desire was to allow overachievement within reason. This was done to allow the other goals a greater chance of affecting the final solution.

The initial district resources budget for special compensation was set at $58,395 (row 1) and each individual job factor (rows 2-13) was set at $167 ($58,395/349) for each scale level. The solution values of the variables

then reflect the relative importance of each variable in view of the constraints and goals used.

The results are as follows:

Variable	Value (in dollars)
X_1	225.10
X_2	167.00
X_3	167.00
X_4	177.00
X_5	167.00
X_6	167.00
X_7	167.00
X_8	205.40
X_9	186.70
X_{10}	119.50
X_{11}	205.40
X_{12}	186.20

These results indicate that variables 1, 4, 8, 9, 11 and 12 were the most important since they exceeded the base value of $167. Variables 2, 3, 5 and 6 equal the base value and variable 10 was of least importance in this formulation.

Four similar models, formulated by changes in various priorities, weighting factors and goals, were tested as part of the study. The result of model testing was a conclusion that a goal programming model was developed which did demonstrate the capability to develop a job factor compensation plan in a public school setting. The model was able to relate goal statements of a prioritized and weighted nature to a series of mathematical

relationships and produce useful output for the decision-making process.

Conclusion

The adequacy of any single set of output is dependent upon environmental and human considerations that are beyond the scope of any model. However, by using a tool such as goal programming, a significant aid is provided to the decision-making process and the consequences of a given set of goals can be evaluated ahead of time.

REFERENCES

1. Berrie, Phillip J., A Linear Programming Model to Optimize the District-Wide Assignment of Teachers, Ph.D. Thesis, The University of Iowa, 1972.

2. Bruno, James E., How To Develop Salary Schedules Which Reflect School District Priorities and Objectives, Swarthmore, Pennsylvania: A.C. Croft, Inc., 1969.

3. Bruno, James E., "An Alternative to the Fixed Step Salary Schedule", Educational Administration Quarterly, Vol. VI, No. 1, Winter, 1970, pp. 26-46.

4. Charnes, A. and W.W. Cooper, Management Models and Industrial Applications of Linear Programming, New York: John Wiley and Sons, Inc., 1961.

5. _____, et al. "A Goal Programming Model for Media Planning," Management Science, Vol. 14, No. 8 (April 1968), pp. 423-30.

6. _____, et. al. "Note on an Application of a Goal Programming Model for Media Planning," Management Science, Vol. 14, No. 8 (April 1968), pp. 431-36.

7. _____ and Ferguson, R. "Optimal Estimation of Executive Compensation by Linear Programming," Management Science, Vol. 1, No. 2, (January 1955) pp. 138-51.

8. Gunderson, James O., A Goal Programming Model to Determine a Job Factor Compensation Plan in a Public School Setting, Ph.D. Thesis, The University of Iowa, 1975.

9. Hartley, Allan C., A Linear Programming Model for the Determination of University Faculty Salaries, Ph.D. Thesis, The University of Iowa, 1973.

10. Ijiri, Y., Management Goals and Accounting for Control, Chicago: Rand McNally, 1965.

11. Keown, A.J. and S.M. Lee, "Integer Goal Programming Model for Urban Planning," 48th Joint National Meeting ORSA/TIMS, Las Vega, Nevada, November 1975.

12. Lee, S.M., Goal Programming for Decision Analysis, Philadelphia: Auerbauch Publishers, Inc., 1972.

13. Matzke, Orville R., A Linear Programming Model to Optimize Various Objective Functions of a Foundation Type State Aid Program, Ph.D. Thesis, The University of Iowa, 1971.

14. Ontjes, Robert L., A Linear Programming Model for Assigning Students to Attendance Centers, Ph.D. Thesis, The University of Iowa, 1971.

15. Stimson, David H. and Ronald P. Thompson, "Linear Programming, Busing and Educational Administration", Socio-Economic Planning Sciences, Vol. 8, No. 4 (August 1974) pp. 195-206.